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# **Antenna Coupling/Isolation Analysis: Generation to 3-D**

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### 14. ABSTRACT

This report summarizes a generalized formulation for three-dimensional antenna coupling and isolation problems. The formulation is developed for a generic transmit/receive configuration. The formulation is based on representing the boundary conditions arising in the problem by using integro-differential representations of Maxwell's equations. To streamline the derivations, integral operators are introduced. The resulting system of coupled equations for the electric and magnetic fields is solved using the Method of Moments (MM) technique. Specifically, the Galerkin variant of this technique is adopted here, resulting in a symmetric system matrix for the problem.

The formulation discussed here is specialized to the two-domain case. This refers to problems where the antenna or array is embedded in a conducting surface constituting one domain. The second domain is an intervening region that is penetrable. Depending on the nature of the penetrable region, it provides the isolation or coupling between the transmit or receive antennas and arrays.

The resulting generalized formulation has been implemented with computer software that is an adaptation of the MM-based CARLOS code. Discussion of the associated numerical implementation and software are documented elsewhere and omitted from this report.

### 15. SUBJECT TERMS

Theoretical formulation for 3-D antenna analysis; Method of Moments; Antenna coupling

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# ANTENNA COUPLING/ISOLATION ANALYSIS: GENERATION TO 3-D

### 1. BACKGROUND

In an earlier report [1], we discussed the issue of antenna coupling/isolation in the presence of layered materials using a two dimensional (2-D) theoretical formulation. Such a formulation can provide general trends for antenna isolation. In particular, one can study effects of discontinuities and tapers of the material layers used for isolation. Because the formulation was 2-D, the numerical implementation results in algorithms that are inherently computationally fast and require minimum computer memory.

After general trends are established with a 2-D simulation, a more accurate calculation requires a 3-D analysis. Such an analysis is important when cross polarization effects are present. Unfortunately, 3-D simulations are much slower and require substantially more computer resources. The following summarizes the generalization of the 2-D formulation to the 3-D case. For clarity and ease of reference, we adhere to the notational scheme in [1] as much as possible.

### 2. STATEMENT OF PROBLEM

Let us consider a radiating and a receiving system embedded in a surface as depicted in Figure 1. Again for generality each of these systems may consist of individual or multiple antenna elements. The surface between the elements can be conducting or consist of single- or multi-layered dielectric (and/or magnetic) coatings on a conducting surface. The electrical characteristics of this surface strongly dictate the isolation (or coupling) of the transmitting and receiving antenna elements.

As in the 2-D formulation, the outer surface of these coatings can be smooth or discontinuous including corrugated conducting surfaces. Such a surface provides a reactive boundary that leads to surface wave guidance. With an appropriate choice of profiles, surfaces can be designed to have stop-band or pass-band properties. The antenna/isolation material regions can be flat or curved. In most cases in practice the array surface is flat. The intervening region between the transmitting or receiving array may be flat or curved.

To provide a framework for the subsequent discussion, we depict the antenna isolation problem in Figure 2. The discontinuities and terminations associated with the antenna apertures and transitions between the coatings can be either electrically large or small with respect to wavelength. The transmitting and receiving antennas/arrays may have different footprints as well.

The problem depicted in Figure 2 is sometimes called a two-domain problem. The antenna or array, embedded in the conducting surface, is designated as one domain (or region). Here it is with bounded by the surfaces  $S_0$  and  $S_2$ . The dielectric region is designated as the second domain (or region) bounded by the surface  $S_2$  and with the surface  $S_1$  at the free space/dielectric interface. Clearly, more complex arrangements are possible. These can also be treated with the following analysis. They do not introduce any new theoretical or mathematical issues other than more notational complexity in the derivations. In the interest of clarity and simplicity we will forego such generalizations.

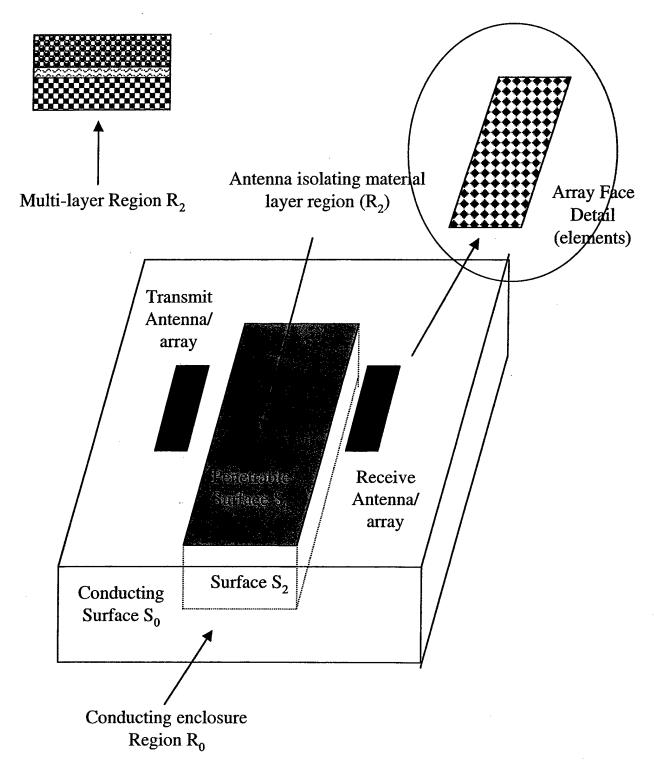


Figure 1: Generic Transmit/Receive Configuration

Signature Technology Office Code 5050 Naval Research Laboratory  $R_1$  (free space region:  $\varepsilon_1$ ,  $\mu_1$ )

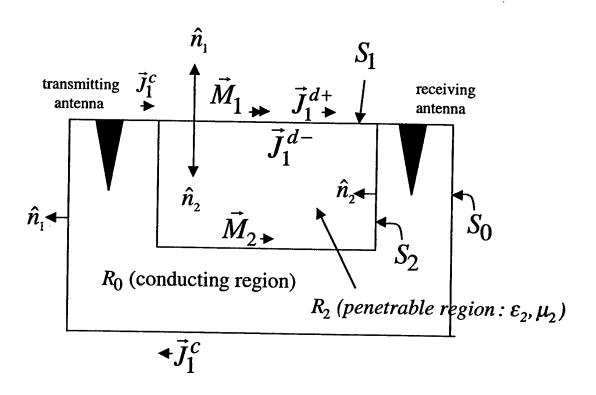


Figure 2: Detail of Two-Region Problem

### 3. 3-D FORMULATION OF THE TWO-DOMAIN PROBLEM

Referring to Figure 2, the transmitting and receiving antennas or arrays are embedded in the conducting surface  $S_0$ . The conducting region  $R_0$  need not be a solid conductor. Often in practice this is an enclosure containing the antenna connectors, phase shifters, and other electronic components of the array back-plane. As in the earlier 2-D formulation, we assume that all the antenna/array elements themselves are surface embedded. The LID program assumes such elements.

Region  $R_2$  defines the penetrable region with the exterior free space boundary of  $S_1$  and the remaining boundary being surface  $S_2$ . The dielectric region is characterized by complex permittivity and/or permeability. For the moment to avoid unnecessary notational complexity, we assume that this region is a homogeneous, isotropic layer. The formulation can be, and has been, generalized for the case of multi-layered and tapered media. The CARLOS computer implementation incorporates these generalizations.

Using the notation in Figure 2, the defining equations for the electromagnetic interactions for each of the regions can be written in terms of integral equations written in integral operator form. Topologically they have the same form as the 2-D formulation discussed earlier in [1]. But in this case the spatial coordinate becomes r. The total electric and magnetic fields in the free-space region  $R_1$  are given respectively by:

$$\theta_1(\vec{r})\vec{E}_1(\vec{r}) = -L_1(\vec{J}_1^c + \vec{J}_1^{d+})(\vec{r}) + K_1\vec{M}_1^+(\vec{r}), \qquad \vec{r} \in R_1$$
 (1)

$$\theta_1(\vec{r})\vec{H}_1(\vec{r}) = -K_1(\vec{J}_1^c + \vec{J}_1^{d+})(\vec{r}) - \frac{1}{\eta_1^2} L_1 \vec{M}_1^+(\vec{r}), \qquad \vec{r} \in R_1$$
 (2)

The vector quantities or functionals  $\vec{J}(\vec{r})$  and  $\vec{M}(\vec{r})$  refer to the electric and magnetic surface currents on the respective boundaries that separate the respective regions, depicted in Figure 2. Fundamentally these are the Kirchhoff loop currents of circuit theory, generalized here to the vector field problem. The currents have a spatial dependence r where r is the spherical coordinate for a 3-D geometry.

The subscripts on the current terms refer to the surfaces forming the various boundaries. The superscripts refer to the type of the boundary (i.e., conducting or dielectric) on which the currents reside. For example,  $\vec{J}_1^c$  refers to the electric currents on a conducting surface. Likewise,  $\vec{J}_1^{d+}$  refers to the currents on the upper surface of the dielectric region forming the boundary with the free space region.

In like manner the total fields in the penetrable region R<sub>2</sub> can be written as:

$$\theta_2(\vec{r})\vec{E}_2(\vec{r}) = -L_2(\vec{J}_2 + \vec{J}_1^{d-})(\vec{r}) + K_2\vec{M}_1^-(\vec{r}), \qquad \vec{r} \in R_2$$
 (3)

$$\theta_2(\vec{r})\vec{H}_2(\vec{r}) = -K_2(\vec{J}_2 + \vec{J}_1^{d-})(\vec{r}) - \frac{1}{\eta_2^2} L_2 \vec{M}_1^-(\vec{r}), \qquad \vec{r} \in R_2$$
 (4)

The symbol  $\theta(\vec{r})$  denotes the usual Heaviside function on and inside the boundary of region R<sub>1</sub> or R<sub>2</sub>. For region R<sub>1</sub> it is defined as:

$$\theta_{1}(\vec{r}) = \begin{cases} 1 & \vec{r} \in R_{1} \\ 1/2 & \vec{r} \in \partial R_{1} \\ 0 & otherwise \end{cases}$$
 (5)

To achieve notational compactness we have introduced the integro-differential operators L and K in Eqs. (1-4), defined as

$$L_{1}\vec{X}(\vec{r}) = \int_{\partial S_{1}} (j\omega\mu_{1}\vec{X}(\vec{r}) + \frac{j}{\omega\varepsilon_{1}} \nabla\nabla' \cdot \vec{X}(\vec{r}))\Phi_{1}(\vec{r} - \vec{r}')d\vec{r}'$$

$$K_{1}\vec{X}(\vec{r}) = \int_{\partial S_{1}} \vec{X}(\vec{r}') \times \nabla\Phi_{1}(\vec{r} - \vec{r}')d\vec{r}'$$
(6)

$$K_1 \vec{X}(\vec{r}) = \int_{\mathcal{S}_1} \vec{X}(\vec{r}') \times \nabla \Phi_1(\vec{r} - \vec{r}') d\vec{r}'$$
(7)

where the vector functional  $\vec{X}(\vec{r})$  refers to either the electric  $\vec{J}(\vec{r})$  or the magnetic  $\vec{M}(\vec{r})$ currents, respectively.

For the 3-D case, the Green's function for region R<sub>1</sub> (i.e., the free-space region) is:

$$\Phi_1(\vec{r} - \vec{r}') = \frac{1}{4\pi} |\vec{r} - \vec{r}'|^{-1} exp[-jk|\vec{r} - \vec{r}'|]. \tag{8}$$

where  $k_1$  is the wave number in region  $R_1$ .

For the penetrable (dielectric) region R<sub>2</sub>, the corresponding Green's function is defined in terms of the permittivity and/or permeability and the wave number of that region. The integrals are over the surfaces or boundaries between the regions. The integrals in the L and K operators are defined in the Cauchy principal value sense.

The boundary conditions on each side of the penetrable surface requires the continuity of the tangential fields, namely:

$$\vec{E}_{1}(\vec{r})_{tanS_{1}} = \vec{E}_{2}(\vec{r})_{tanS_{1}} \tag{9}$$

$$\vec{H}_{1}(\vec{r})_{tan\,S_{1}} = \vec{H}_{2}(\vec{r})_{tan\,S_{1}} \tag{10}$$

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The tangential electric fields vanish on the conducting surfaces yielding

$$\vec{E}_1(\vec{r})_{tan\,S_0} = 0 \tag{11}$$

$$\vec{E}_1(\vec{r})_{tan\,S_2} = 0 \tag{12}$$

Letting  $\vec{M}_1 = \vec{M}_1^+ = -\vec{M}_1^-$  and  $\vec{J}_1^d = \vec{J}_1^{d+} = -\vec{J}_1^{d-}$  on the dielectric/free-space boundary, one obtains six coupled integral equations for the four unknown currents on all the boundaries. For points on the dielectric/free-space boundary, we obtain

$$\left[L_{1}\vec{J}_{1}^{c}(\vec{r}) + (L_{1} + L_{2})\vec{J}_{1}^{d}(\vec{r}) - L_{2}\vec{J}_{2}(\vec{r}) - (K_{1} + K_{2})\vec{M}_{1}(\vec{r})\right]_{tan S_{1}^{d}} = 0$$
(13)

$$\left[K_{1}\vec{J}_{1}^{c}(\vec{r}) + (K_{1} + K_{2})\vec{J}_{1}^{d}(\vec{r}) - K_{2}\vec{J}_{2}(\vec{r}) - (\frac{1}{\eta_{1}^{2}}L_{1} + \frac{1}{\eta_{2}^{2}}L_{2})\vec{M}_{1}(\vec{r})\right]_{tan S} = 0$$
 (14)

On the exterior conducting boundary, electric and magnetic field integral equations (EFIE and MFIE) are respectively,

$$\left[L_1(\vec{J}_1^c + \vec{J}_1^d)(\vec{r}) - K_1\vec{M}_1(\vec{r})\right]_{tanS.} = 0$$
 (15)

$$\left[\frac{\vec{J}_{1}^{c}(\vec{r})}{2} + \overline{n}_{1} \times K_{1}(\vec{J}_{1}^{c} + \vec{J}_{1}^{d})(\vec{r}) + \frac{1}{\eta_{1}^{2}} \overline{n}_{1} \times L_{1} \vec{M}_{1}(\vec{r})\right]_{tan S_{0}} = 0$$
 (16)

Finally, on the interior conducting boundary, the EFIE and MFIE formulations are respectively,

$$\left[L_2(\vec{J}_2 - \vec{J}_1^d)(\vec{r}) + K_2\vec{M}_1(\vec{r})\right]_{tan S_2} = 0$$
 (17)

$$\left[\frac{\vec{J}_{2}(\vec{r})}{2} + \overline{n}_{2} \times K_{2}(\vec{J}_{2} - \vec{J}_{1}^{d})(\vec{r}) - \frac{1}{\eta_{2}^{2}} \overline{n}_{2} \times L_{2} \vec{M}_{1}(\vec{r})\right]_{tan S_{2}} = 0$$
 (18)

The imposition of the boundary conditions results in six equations for the four unknown currents on the boundaries. Subsequently only four equations will be used, i.e. those associated with the PMCHW formulation (after Poggio, Miller, Chu, Harrington and Wu), namely Eqs. (13)-(14), and the E-field boundary condition on the conducting surfaces, i.e., Eqs. (15) and (17). [2,3]

configuration was primarily chosen to introduce the notation required to carry out the above derivation.

The principal restriction so far is that the problem consists of two domains: one conducting and the other penetrable and homogeneous. As noted before, this restriction was imposed to streamline the foregoing derivation. A more general derivation is possible that not removes this restriction. The CARLOS computer implementation involves the more general case, i.e., where Region R<sub>2</sub> is multi-layered. A further generalization allows the antenna/array elements to be embedded within the penetrable (dielectric) medium itself. To achieve RCS suppression of array apertures often requires specialized antenna windows. In such cases the foregoing generalized analysis option is required.

### 4. SOLUTION OF THE TWO-DOMAIN PROBLEM

There are a number of ways of solving the derived system of equations. In this report we confine ourselves to the method of moments (MM) technique. We use the Galerkin variant of this method. This approach has proven to be computationally robust and highly flexible in handling a great variety of problems. [4]

Following the Galerkin MM technique, one discretizes the surfaces in Figure 2 with flat triangular facets. The currents on each facet are expanded in terms of roof-top function s at each interior edge on a surface. Half roof-tops are used for junctions between two surfaces intersecting where for instance a conducting and a dielectric surface meet. These "junction" functions allow the Kirchhoff continuity condition to be met in the numerical implementation. A detailed description of these functions and the associated junction treatments is given in Section 2.6 of [5]. Next one expands the unknown currents on the conducting and dielectric boundaries via the set of expansion functions. Here we choose triangle functions of the Rao-Wilton-Glisson (RWG) type.

The expansion of the electric and magnetic surface currents on each facet can be written explicitly. For example,

$$\vec{J}(t) \approx \sum I_n \vec{f}_n(\vec{r}) \tag{19}$$

$$\vec{J}(t) \approx \sum_{n} I_{n} \vec{f}_{n}(\vec{r})$$

$$\vec{M}(t) \approx -\eta_{0} \sum_{n} K_{n} \vec{f}_{n}(\vec{r})$$
(19)

where  $\vec{f}_n(\vec{r})$  are roof-top functions, and  $I_n$  and  $K_n$  are the unknown coefficients of the expansions. Since the foregoing expansions are over a finite number of terms, the current representations are approximate. However, in practice if the facet surface discretization is on the order of  $\lambda 10$  per linear dimension, convergent results are obtained.

The current expansions are substituted into the integral equations and the Galerkin testing procedure is carried out in Eqs. (13)-(15) and (17). The details are discussed in [5]. The Galerkin procedure yields a system of equations linear in the unknown coefficients of the electric and magnetic currents  $I_n$  and  $K_n$  on the surfaces. This system is customarily written in matrix form as:

$$ZI = V (21)$$

The Z matrix contains all the electromagnetic interactions of the transmitting and receiving antennas in the presence of the penetrable region R<sub>2</sub> between them. It should be noted that these interactions include both the radiated field coupling between the antennas as well as the surface current (or conduction) coupling. For closely collocated transmitting and receiving systems, the latter is dominant and depends on the electrical characteristics of the surface (ground plane) between them.

The general form of the Z matrix is:

$$Z = \begin{bmatrix} L(S_{1}, S_{1}; R_{1}) \\ +L(S_{1}, S_{1}; R_{2}) \end{pmatrix} \begin{pmatrix} -K(S_{1}, S_{1}; R_{1}) \\ -K(S_{1}, S_{1}; R_{2}) \end{pmatrix} & L(S_{1}^{d}, S_{1}^{c}; R_{1}) & -L(S_{1}^{d}, S_{2}; R_{2}) \\ K(S_{1}, S_{1}; R_{1}) + \\ K(S_{1}, S_{1}; R_{2}) \end{pmatrix} \begin{pmatrix} \frac{\varepsilon_{1r}}{\mu_{1r}} L(S_{1}, S_{1}; R_{1}) + \\ \frac{\varepsilon_{2r}}{\mu_{2r}} L(S_{1}, S_{1}; R_{2}) \end{pmatrix} & K(S_{1}^{d}, S_{1}^{c}; R_{1}) & -K(S_{1}^{d}, S_{2}; R_{2}) \\ L(S_{0}, S_{1}; R_{1}) & -K(S_{0}, S_{1}; R_{1}) & L(S_{0}, S_{0}; R_{1}) & 0 \\ -L(S_{2}, S_{1}; R_{1}) & K(S_{2}, S_{1}; R_{2}) & 0 & L(S_{2}, S_{2}; R_{2}) \end{bmatrix}$$

$$(22)$$

The submatrix blocks of Z were obtained by forming the vector interproduct of the integral operators with the testing function. In the case of the Galerkin implementation of the MM technique, the latter is the complex conjugate of the expansion functions used for the electric and magnetic currents. Thus for example, the block  $L(S_1, S_2; R)$  is obtained by forming the interproduct of the integral operator L on  $S_2$  and the testing function on  $S_1$ . Further details of this procedure are given in [5].

The column vector I in Eq. (21) represents the coefficients of unknown currents on the conducting and dielectric boundaries. Specifically, it is given as:

$$I = \begin{vmatrix} J_1^d \\ M_1 \\ J_1^c \\ J_2 \end{vmatrix}$$
 (23)

Note subparts of this column vector are the electric and magnetic unknown currents on the various surfaces. The superscripts designate the latter. The column vector V in Eq. (21) denotes the aperture excitation of the transmitting antenna, specifically the electric and magnetic fields in the transmitting aperture (or apertures). It is given as:

$$V = -\begin{bmatrix} E(S_1) \\ H(S_1) \\ E(S_1) \\ 0 \end{bmatrix}$$
 (24)

The elements of V are given known quantities. For example, if the antenna aperture is embedded in the conducting surface  $S_0$ , the quantity  $E(S_0)$  represents the electric field excitation (or voltage impressed) on the aperture. In this case the other entries of the array

Signature Technology Office Code 5050 Naval Research Laboratory are zero. Knowing the aperture excitation, the foregoing matrix system in Eq. (21) can be solved for the unknown currents either by LU decomposition or an iterative technique.

## 5. NUMERICAL IMPLEMENTATION

The 3-D formulation summarized above, has been implemented numerically computer using an adaptation of the CARLOS code. [5] Using this code, detailed simulations can be carried out to quantify the coupling between the transmitting and receiving antennas. In addition, the software allows the distribution of near and ultra-near fields to be computed at any point in the proximity of antennas. If desired, the latter fields can be mapped into the frequency-wave vector domain to elucidate the coupling contributions associated with the radiated and evanescent fields of the transmitting antenna. Earlier, such mapping was carried out for scattering problems. [6-8] The implementing software applies equally well to the present radiation/antenna problems.

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